



Measurement of charged-particle multiplicity distributions and their H_q moments in hadronic Z decays at LEP

L3 Collaboration

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Abstract

The charged-particle multiplicity distribution is measured for all hadronic events as well as for light-quark and b-quark events produced in e^+e^- collisions at the Z pole. Moments of the charged-particle multiplicity distributions are calculated. The H_q moments of the multiplicity distributions are studied, and their quasi-oscillations as a function of the rank of the moment are investigated.

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1. Introduction

Since quarks and gluons are not observed directly, the understanding of the hadronization process whereby a quark–gluon system evolves to hadrons is of importance and provides a tool for studying the quark–gluon system itself. One of the most basic characteristics of the resulting hadronic system is the distribution of the number of hadrons produced.

Assuming local parton–hadron duality (LPHD) [1], characteristics of the charged-particle multiplicity distribution are directly related to the characteristics of the corresponding parton distributions. The parton distributions are calculable using perturbative quantum chromo-dynamics (pQCD). In particular, the dependence on the center-of-mass energy, \sqrt{s} , of the mean, $\langle n \rangle$, of the charged-particle multiplicity is an important test of pQCD. Since these calculations are only valid for light quarks, a separate measurement for light quarks is of interest.

In this Letter, the charged-particle multiplicity distributions of hadronic decays of the Z boson are measured for b- and for light-quark (u, d, s and c) events as well as for all events. From these distributions moments are calculated, which characterize the shape of the distributions.

The shape of the charged-particle multiplicity distribution is a fundamental tool in the study of particle production. Independent emission of single particles leads to a Poissonian multiplicity distribution. Deviations from this shape, therefore, reveal correlations [2]. To study the shape, we use the normalized factorial moments. In terms of the multiplicity distribution, $P(n)$, the normalized factorial moment of rank q is defined by

$$F_q = \frac{\sum_{n=q}^{\infty} n(n-1)\cdots(n-q+1)P(n)}{(\sum_{n=1}^{\infty} nP(n))^q}. \quad (1)$$

It reflects correlations in the production of up to q particles. If the particle distribution is Poissonian, all F_q are equal to unity. If the particles are correlated, the distribution is broader and the F_q are greater than unity. If the particles are anti-correlated, the distribution is narrower and the F_q are less than unity.

Normalized factorial cumulants, K_q , obtained from the normalized factorial moments by

$$K_q = F_q - \sum_{m=1}^{q-1} \frac{(q-1)!}{m!(q-m-1)!} K_{q-m} F_m, \quad (2)$$

measure the genuine correlations between q particles, i.e., q -particle correlations which are not a consequence of correlations among fewer than q particles.

Since $|K_q|$ and F_q both increase rapidly with q , it is useful to define the H_q moments,

$$H_q = \frac{K_q}{F_q}, \quad (3)$$

which have the same order of magnitude over a large range of q .

The shape of the charged-particle multiplicity distribution analyzed in terms of the H_q was found to reveal quasi-oscillations [3–6], when plotted versus the rank q , in e^+e^- , as well as hadron–hadron, hadron–ion and ion–ion interactions. In e^+e^- annihilation,

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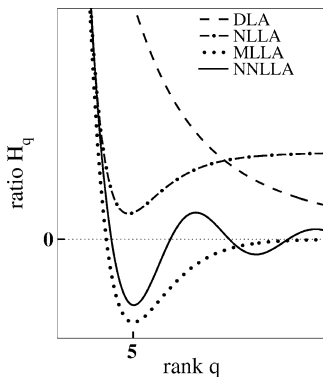


Fig. 1. Qualitative behavior of H_q as a function of q for various approximations of perturbative QCD [3,8].

this result was interpreted [5,7] in terms of pQCD, from which the H_q of the parton multiplicity distribution were calculated [3,8]. The expected behavior of H_q vs. q is quite sensitive to the approximation used, as is illustrated qualitatively in Fig. 1 for the double logarithm approximation (DLA), the modified leading logarithm approximation (MLLA), the next-to-leading logarithm approximation (NLLA), and the next-to-next-to-leading logarithm approximation (NNLLA). In the NNLLA a negative first minimum is expected near $q = 5$ and quasi-oscillations about zero are expected for larger values of q .

According to the LPHD hypothesis, hadronization does not distort the shape of the multiplicity distribution. If this is valid, the same shape may be expected for the charged-particle multiplicity distribution as for the parton multiplicity distribution.

2. Experimental procedures

2.1. Event selection

This analysis is based on 1.5 million hadronic events collected by the L3 detector [9] at LEP in the years 1994 and 1995 at the Z pole.

Events are selected in a two-step procedure [10]. First, at least 15 calorimetric clusters of at least 100 MeV are required in order to reduce background from the $e^+e^- \rightarrow \tau^+\tau^-$ process. Hadronic events from the process $e^+e^- \rightarrow q\bar{q}$ are then selected by requiring small energy imbalance both along and transverse to the beam direction.

The second step is the selection of charged tracks measured in the central tracker and the silicon microvertex detector. A number of quality cuts are used to select well-measured tracks. Further, the thrust direction calculated from the charged tracks is required to lie within the full acceptance of the central tracker. No selection specifically rejects or selects tracks from long-lived neutral particles. The track selection efficiency, determined from Monte Carlo, is about 75%. The resulting data sample corresponds to approximately one million selected hadronic events, and has a purity of about 99.8%.

To correct for detector acceptances and inefficiencies, we make use of the JETSET 7.4 [11] parton shower Monte Carlo program, tuned using L3 data. Events are generated, passed through the L3 detector simulation program [12], and further subjected to time-dependent detector effects. Then they are reconstructed and the events and tracks are selected in the same way as the data. For systematic studies we also use events generated by ARIADNE 4.2 [13]. For comparisons with the data we use HERWIG 5.9 [14] as well as JETSET.

To select b- and udsc-quark enhanced samples, we use the full three-dimensional information on tracks from the central tracker to calculate for each track the probability that it originated at the primary vertex [15]. We select b- and udsc-quark samples with purities of about 96% and 93% and efficiencies of about 38% and 96%, respectively.

2.2. Unfolding

The resulting multiplicity distributions are fully corrected for detector resolution using an iterative Bayesian unfolding method [16]. The detector and generator level Monte Carlo events are used to construct a matrix $R(n_{\text{det}}, n)$ which represents the probability that n_{det} tracks would be detected if n charged particles were produced. A distribution, $P_0(n)$, is assumed for n . For this P_0 , the distribution expected in the detector is $P_0^{\text{det}}(n_{\text{det}}) = \sum_n R(n_{\text{det}}, n)P_0(n)$. This is compared to the actual distribution of the raw data, and, making use of Bayes' theorem, an improved multiplicity distribution is calculated, which replaces $P_0(n)$ in the above expression. This process is repeated iteratively until satisfactory agreement between the expected and actual raw data distribution is found. In

practice, this occurs after the second iteration if the JETSET multiplicity distribution is chosen as $P_0(n)$.

In addition, corrections are made for efficiency and acceptance of the event selection, initial state radiation, and K_S^0 and Λ decays. Furthermore, the distributions for the b- and udsc-enhanced samples are corrected for the purity of the flavor selection.

The unfolding method gives [16] an estimate of the covariance matrix of the unfolded distribution. This matrix, combined with the uncertainties on the corrections mentioned above, is used to determine the uncertainties on the moments of the multiplicity distribution for the all- and udsc-flavor cases. When the statistics is too small, as in the b-flavor case, the uncertainty on the estimate of the covariance matrix is large. In this case we use a Monte Carlo method. Many Monte Carlo variations of the raw data multiplicity distribution are made, choosing the number of events at each multiplicity from a Poisson distribution having as mean the observed number of events. These Monte Carlo distributions are then analysed in the same way as the data distribution. The uncertainty on a moment is determined from the spread in values of the moments of the Monte Carlo distributions. For the high-statistics cases, both methods agree.

2.3. Systematic uncertainties

The following sources of systematic uncertainty are investigated:

Selection The value of each cut used in the event selection is varied independently over a reasonable range around the default value and the resulting fully corrected distributions, together with their covariance matrices, determined, and from them the moments of the multiplicity distribution. For each multiplicity, as well as for each multiplicity moment, we assign a systematic uncertainty of half of the maximum difference between the new values. The same procedure is followed for the track selection and flavor tagging. For flavor tagging there is an additional contribution due to an uncertainty of 2.5% in the purity of the resulting sample, which accounts for the different response of the tagging algorithm to data and Monte Carlo.

Monte Carlo uncertainties The analysis is repeated using ARIADNE instead of JETSET to determine the corrections and the unfolding matrix. The difference between the two results is taken as the systematic uncertainty. Further, the c- and b-quark fragmentation parameters, ϵ_c and ϵ_b , are varied. Also, the strangeness suppression parameter is varied by an amount consistent with the measured K_S^0 production rate [17]. In each case, half the difference between the results using the two parameter values is taken as the systematic uncertainty.

Unfolding method Three contributions are determined: first, ARIADNE is used to derive the initial distribution. Secondly, the analysis is repeated using a different number of iterations in the unfolding. Finally, the detector level multiplicity distribution of events generated by ARIADNE is unfolded using the response matrix, $R(n_{\text{det}}, n)$, determined using JETSET events. In each case, the difference from the default value is taken as the systematic uncertainty.

Background The background of about 0.2% is mostly from two-photon processes. We take as a systematic uncertainty the effect of twice the amount of estimated background.

The contributions from each of these sources are added in quadrature. The track selection contributes the dominant part of the total systematic uncertainty when all events are used, while the flavor-tagging purity uncertainty dominates that of the udsc sample.

Table 1

Contribution of the various sources of systematic uncertainty to the measurement of the mean charged-particle multiplicity, $\langle n \rangle$

Source	full	udsc	b
Event selection	0.005	0.006	0.004
Track selection	0.090	0.080	0.116
Tagging cuts		0.018	0.021
Tagging purity		0.185	0.126
MC modeling	0.032	0.031	0.040
Unfolding	0.034	0.034	0.043
Background	0.024	0.024	0.023
γ conversion	0.039	0.039	0.039
Total	0.11	0.21	0.19

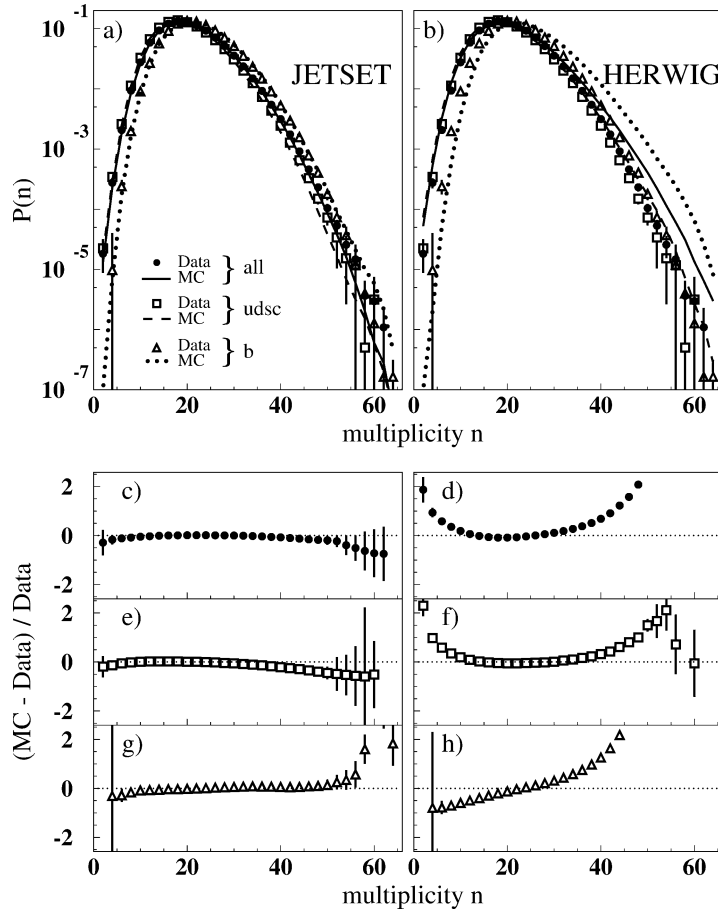


Fig. 2. Charged-particle multiplicity distribution for all, udsc-, and b-quark events compared to the expectations of (a), (c), (e), (g) JETSET and (b), (d), (f), (h) HERWIG. The error bars include both statistical and systematic uncertainties.

For the b-quark sample, these two contributions are about equal.

In addition, the accuracy of the simulation of the rate of photon conversion is considered. This is found to be about 15% smaller than in data [10] and is assigned as a systematic uncertainty on $\langle n \rangle$. It is found to be negligible for the other moments. Breakdowns of the systematic uncertainties on $\langle n \rangle$ are shown in Table 1.

3. Results

3.1. Charged-particle multiplicity distributions

Charged-particle multiplicity distributions are measured both including and excluding K_S^0 and Λ decay

products.⁷ Fig. 2 shows the charged-particle multiplicity distribution including K_S^0 and Λ decay products for the full, udsc- and b-quark samples. All distributions agree rather well with JETSET, but in all cases HERWIG gives a poor description of the data, as is seen in Fig. 2(a) and 2(b).

From these distributions various moments of the charged-particle multiplicity distribution are calculated. The results are summarized in Table 2. The mean multiplicity including K_S^0 and Λ decay products

⁷ Note that Σ^- , Ξ^- and Ω^- have only one charged particle among their decay products apart from those produced in Λ decay, and Σ^0 and Ξ^0 have none. Thus including or not the decay products of these baryons does not affect the charged multiplicity except through the Λ decay.

Table 2

Moments of the charged-particle multiplicity distribution for all, udsc-, and b-quark events. The first uncertainty is statistical, the second systematic

Moments	Without K_S^0 and Λ decay	With K_S^0 and Λ decay
<i>All events</i>		
$\langle n \rangle$	$18.63 \pm 0.01 \pm 0.11$	$20.46 \pm 0.01 \pm 0.11$
$\langle n^2 \rangle$	$381.7 \pm 0.3 \pm 4.4$	$457.7 \pm 0.3 \pm 4.9$
$\langle n^3 \rangle \times 10^{-2}$	$85.2 \pm 0.1 \pm 1.5$	$111.1 \pm 0.1 \pm 1.8$
$\langle n^4 \rangle \times 10^{-3}$	$205.9 \pm 0.4 \pm 5.1$	$290.6 \pm 0.5 \pm 6.5$
$D = \sqrt{\langle (n - \langle n \rangle)^2 \rangle}$	$5.888 \pm 0.005 \pm 0.051$	$6.244 \pm 0.005 \pm 0.051$
$S = \langle (n - \langle n \rangle)^3 \rangle / D^3$	$0.596 \pm 0.004 \pm 0.010$	$0.600 \pm 0.004 \pm 0.010$
$K = \langle (n - \langle n \rangle)^4 \rangle / D^4 - 3$	$0.51 \pm 0.01 \pm 0.04$	$0.49 \pm 0.01 \pm 0.03$
$\langle n \rangle / D$	$3.164 \pm 0.002 \pm 0.016$	$3.277 \pm 0.002 \pm 0.016$
$F_2 = \langle n(n-1) \rangle / \langle n \rangle^2$	$1.0461 \pm 0.0002 \pm 0.0040$	$1.0441 \pm 0.0001 \pm 0.0034$
<i>udsc-quark events</i>		
$\langle n \rangle$	$18.07 \pm 0.01 \pm 0.21$	$19.88 \pm 0.01 \pm 0.21$
$\langle n^2 \rangle$	$340.0 \pm 0.3 \pm 8.4$	$432.4 \pm 0.4 \pm 9.2$
$\langle n^3 \rangle \times 10^{-2}$	$78.3 \pm 0.1 \pm 2.7$	$102.2 \pm 0.1 \pm 3.3$
$\langle n^4 \rangle \times 10^{-3}$	$184.4 \pm 0.4 \pm 8.6$	$260.7 \pm 0.5 \pm 11.1$
$D = \sqrt{\langle (n - \langle n \rangle)^2 \rangle}$	$5.769 \pm 0.007 \pm 0.071$	$6.111 \pm 0.007 \pm 0.071$
$S = \langle (n - \langle n \rangle)^3 \rangle / D^3$	$0.613 \pm 0.005 \pm 0.014$	$0.617 \pm 0.005 \pm 0.012$
$K = \langle (n - \langle n \rangle)^4 \rangle / D^4 - 3$	$0.54 \pm 0.02 \pm 0.06$	$0.53 \pm 0.02 \pm 0.05$
$\langle n \rangle / D$	$3.133 \pm 0.003 \pm 0.020$	$3.252 \pm 0.003 \pm 0.020$
$F_2 = \langle n(n-1) \rangle / \langle n \rangle^2$	$1.0464 \pm 0.0002 \pm 0.0045$	$1.0441 \pm 0.0002 \pm 0.0038$
<i>b-quark events</i>		
$\langle n \rangle$	$20.51 \pm 0.02 \pm 0.19$	$22.45 \pm 0.03 \pm 0.19$
$\langle n^2 \rangle$	$453.9 \pm 1.1 \pm 1.8$	$542.0 \pm 1.2 \pm 3.0$
$\langle n^3 \rangle \times 10^{-2}$	$107.9 \pm 0.4 \pm 0.7$	$140.1 \pm 0.5 \pm 1.1$
$\langle n^4 \rangle \times 10^{-3}$	$273.9 \pm 1.5 \pm 1.9$	$385.8 \pm 1.9 \pm 1.7$
$D = \sqrt{\langle (n - \langle n \rangle)^2 \rangle}$	$5.78 \pm 0.01 \pm 0.07$	$6.16 \pm 0.01 \pm 0.07$
$S = \langle (n - \langle n \rangle)^3 \rangle / D^3$	$0.574 \pm 0.017 \pm 0.008$	$0.573 \pm 0.017 \pm 0.007$
$K = \langle (n - \langle n \rangle)^4 \rangle / D^4 - 3$	$0.43 \pm 0.04 \pm 0.04$	$0.42 \pm 0.04 \pm 0.03$
$\langle n \rangle / D$	$3.551 \pm 0.006 \pm 0.055$	$3.645 \pm 0.005 \pm 0.049$
$F_2 = \langle n(n-1) \rangle / \langle n \rangle^2$	$1.0305 \pm 0.0003 \pm 0.0027$	$1.0307 \pm 0.0002 \pm 0.0023$

is consistent with our previous measurements [18,19] and about 0.6 below the world average (21.07 ± 0.11) [17]. The difference in mean multiplicity between the cases of including or not the K_S^0 and Λ decay products is consistent with our measurement of the K^0 and Λ production rates [20] and with the world average [17]. All the moments, with the exception of the dispersion, D , show significant flavor dependence. However, the flavor dependence of F_2 is quite small. F_2 is also quite insensitive to the inclusion or not of K_S^0 and Λ decay products. The difference between the mean

charged-particle multiplicity of the b-quark sample and that of the udsc-quark sample is $2.58 \pm 0.03 \pm 0.08$ when K_S^0 and Λ decay products are included and $2.43 \pm 0.03 \pm 0.08$ otherwise.

3.2. H_q

The H_q are calculated from the unfolded charged-particle multiplicity distributions. Since the H_q are sensitive to low statistics at very high multiplicities, we truncate the multiplicity distribution. The H_q thus

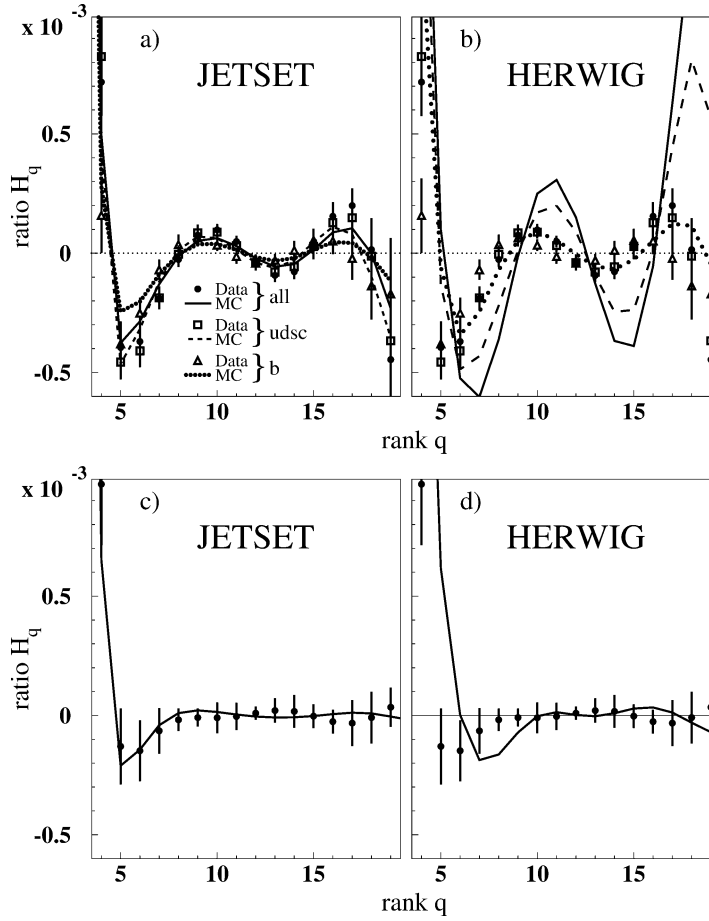


Fig. 3. The H_q of the truncated (a), (b) and non-truncated (c), (d) charged-particle multiplicity distribution compared to the expectations of (a), (c) JETSET and (b), (d) HERWIG. The error bars include both statistical and systematic uncertainties.

obtained are biased estimators of the H_q of the untruncated distribution. This bias increases with stronger truncation, while the statistical uncertainty decreases, which allows a more significant comparison with models. It was suggested [21] that even without this truncation, the H_q may be biased since a natural truncation occurs as a consequence of the finiteness of the sample. The truncation can induce oscillations or increase their size [21]. The truncation also introduces correlations between the H_q , although these are small for low q [10,21,22]. We choose the point of truncation such that multiplicities with relative error on $P(n)$ greater than 50% are rejected. This corresponds, for all multiplicity distributions studied, to about 0.005% of events. For all three samples (full, udsc, and b) the

truncation is at 53 if K_S^0 and Λ decay products are included in the multiplicity and at 49 when they are not. The H_q presented here are calculated from distributions not including these decay products. However, the H_q are insensitive to their inclusion [10].

The H_q of the truncated charged-particle multiplicity distribution from all, udsc- and b-quark events, shown in Fig. 3, have a first negative minimum at $q = 5$ and quasi-oscillations for larger q . They are very similar for the three samples, with only slight differences for the b-quark sample. Similar behavior is seen for JETSET (Fig. 3(c)). Oscillations are also observed for HERWIG (Fig. 3(d)), but they do not agree with those seen in the data. For both data and the Monte Carlo models, truncation at a lower value increases the

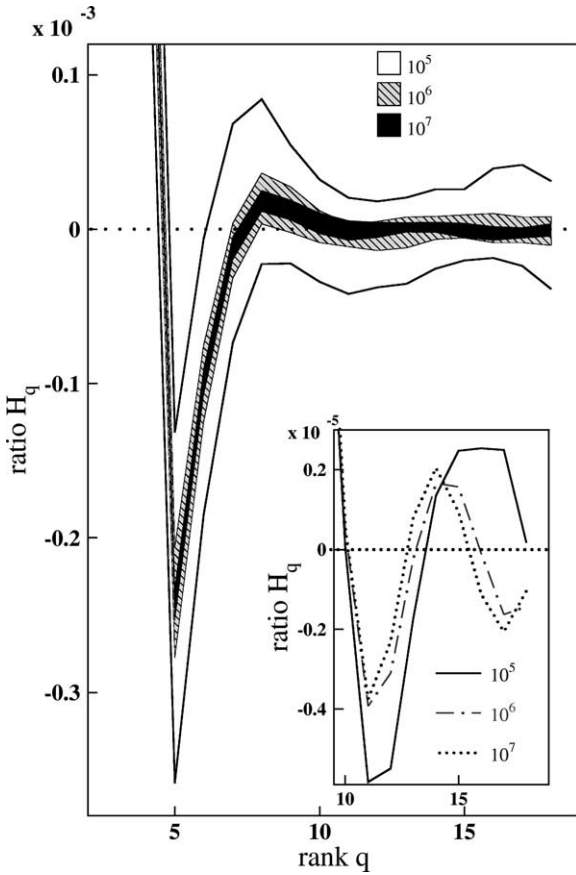


Fig. 4. 1-standard deviation bands of expected H_q of the non-truncated charged-particle multiplicity distribution from PYTHIA for sample sizes of 10^5 , 10^6 and 10^7 . The insert shows the mean H_q of 100 samples of 10^5 , 10^6 and 10^7 PYTHIA events.

depth of the first minimum and the amplitudes of the oscillations, while truncation at a larger value has the opposite effect.

We note that our H_q , based on an order of magnitude greater statistics, agree with the H_q of SLD if we truncate at a value equal to the maximum multiplicity they observed [5].

No truncation, other than that due to the finiteness of the sample, reduces the amplitudes of the oscillations to statistical insignificance, but the minimum at $q = 5$ remains, as is shown in Fig. 3. Again, JETSET agrees well with the data, while HERWIG does not.

To investigate the effect of sample size on the H_q , 100 samples of PYTHIA [23] Monte Carlo events, were generated for sample sizes of 10^5 , 10^6 and 10^7

events, and their H_q determined. Their ± 1 standard deviation bands are shown in Fig. 4. In the insert of Fig. 4 the mean of the values is shown. For large q the values of the H_q depend on the sample size. However, for small q the values of the H_q are stable. In particular, H_5 (the first minimum) changes little with the sample size, giving us confidence that the measured H_5 is robust. Fig. 4 suggests that at least 10^7 events, an order of magnitude beyond the statistics of the present experiment, would be needed to establish the maximum at $q = 8$.

4. Conclusions

The charged particle multiplicity distribution of hadronic Z decay and its moments are measured for light-quark and for b-quark, as well as for all flavor events. The H_q moments of truncated multiplicity distributions, which have smaller statistical uncertainties than those of the full distributions, are plotted versus the rank q . A negative minimum is observed at $q = 5$ followed by quasi-oscillations about zero, which is qualitatively similar to the behavior expected in NNLLA for the H_q moments of the full multiplicity distribution. Since Monte Carlo studies show that these oscillations are magnified, or even created, by truncation of the multiplicity distribution, the H_q are also measured for the untruncated multiplicity distribution. In this case the minimum at $q = 5$, expected in both MLLA and NNLLA, is confirmed. But the oscillations at higher values of q , which are expected only in NNLLA, cannot be confirmed. Previous observations of these oscillations are most likely a consequence of truncation resulting from limited statistics.

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